Instructions: Legibly complete each of the following on lined paper and submit on Gradescope.

1. Consider the following dictionary of predicates (over the integers).

$P(x)$:	x is prime	$E(x)$:	x is even
$O(x)$:	x is odd	$D(x,y)$:	x divides y (or y is divisible by x)

(a) Translate each of the following predicates into English.

$\boxed{*}$ i. $\forall x[P(x) \to O(x)]$

ii. $\exists x[P(x) \wedge E(x)]$

iii. $\forall x[O(x) \leftrightarrow (\neg E(x))]$

$\boxed{*}$ iv. $\forall x[(E(x) \to (D(x,2) \vee O(x))]$

v. $\forall x[D(1,x)]$

vi. $\forall x \exists y[x+y=0]$

$\boxed{*}$ vii. $\exists x \forall y[x+y=0]$

viii. $\forall x \forall y \forall z[(x*y=x*z) \to (y=z)]$

(b) Translate each of the following statements using the dictionary above.

$\boxed{*}$ i. Every integer is either even or odd.

ii. If 3 divides integer x, then x is odd.

$\boxed{*}$ iii. If x is prime and x is greater than two, then x is odd.

iv. An integer is divisible by 6 if and only if it is divisible by both 2 and 3.

$\boxed{*}$ v. For every integer n, if n is strictly greater than two, then for all positive integers x, y, z the equation $x^n + y^n = z^n$ does not hold.

2. Translate the predicate $C((a_n)_{n\in\mathbb{N}}, L)$ about sequences of real numbers into symbolic logic: "For every $\epsilon > 0$ there exists a natural number N such that for all $n \geq N$ we have $|a_n - L| < \epsilon$." What does this predicate define?

3. Create a truth table for each of the following statements, and then compute its disjunctive normal form. Which are tautological? Contradictory? Conditionally true?

(a) $(P \to Q) \to Q$

$\boxed{*}$ (b) $(P \to Q) \to (Q \to P)$

(c) $(Q \leftrightarrow (\neg P)) \vee P$

$\boxed{*}$ (d) $((P \to Q) \wedge P) \leftrightarrow (\neg Q)$

(e) $(P \vee Q) \vee (\neg Q)$

(f) $(P \wedge (\neg Q)) \vee (R \to Q)$

4. Verify each claimed logical equivalence...

i. with a truth table, and

ii. with the algebra of statements.

(a) $(P \to Q) \wedge (P \to R) \equiv P \to (Q \wedge R)$

$\boxed{*}$ (b) $(P \to R) \wedge (Q \to R) \equiv (P \vee Q) \to R$

5. Use natural deduction to show each of the following argument forms is valid. Use the 2-column proof style from class, justifying each step completely via a basic logical equivalence or an argument form from lecture.

$\boxed{*}$ (a) $X \to ((\neg Y) \to (\neg Z))$, $\neg Y$, $Y \vee X$ $\therefore \neg Z$

(b) $X \to (X \to (\neg Y))$, $Y \vee (\neg A)$, X , $W \to A$ $\therefore \neg W$

(c) $X \to Y$, $((\neg X) \vee W) \to A$, $(\neg Y) \wedge Z$ $\therefore A \vee B$

$\boxed{*}$ (d) $(X \to Y) \wedge A$, $(Y \to Z) \wedge B$, $(X \to Z) \to ((X \to Y) \to W)$ $\therefore W$

(e) $(X \wedge Y) \vee (Z \wedge W)$, $(X \wedge Y) \to A$, $(\neg A) \wedge B$, $Z \to (C \wedge D)$ $\therefore C$

(f) $\neg(X \wedge Y)$, X , $Y \vee Z$ $\therefore Z$

(g) $X \wedge (Y \vee Z)$, $\neg(X \wedge Y)$, $(Z \wedge X) \to W$ $\therefore W$

$\boxed{*}$ (h) X , $(Y \to X) \to Z$ $\therefore X \wedge Z$

(i) $X \to Z$, $Y \to Z$ $\therefore (X \vee Y) \to Z$

(j) $X \to (Y \vee Z)$, $\neg Y$ $\therefore X \to Z$

(k) $\neg(X \vee (\neg X))$ $\therefore Y$

6. Is the argument below valid? Give a complete (rigorous!) justification for your answer.

    1. If the dog is bad or does not take a walk, then the dog cannot have a treat.
    2. The dog is not sad.
    3. If the dog is happy, then the dog can have a treat.
    4. Thus, the dog is good.